Energy taxes and endogenous technological change

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Abstract

This paper studies the effects of a tax on energy use in a growth model where market structure is endogenous and jointly determined with the rate of technological change. Because this economy does not exhibit the scale effect (a positive relation between TFP growth and aggregate R&D), the tax has no effect on the steady-state growth rate. It has, however, important transitional effects that give rise to surprising results. Specifically, under the plausible assumption that energy demand is inelastic, there exists a hump-shaped relation between the energy tax and welfare. This shape stems from the fact that the reallocation of resources from energy production to manufacturing triggers a *temporary acceleration* of TFP growth that generates a ✓-shaped time profile of consumption. If endogenous technological change raises consumption sufficiently fast and by a sufficient amount in the long run, the tax raises welfare despite the fact that – in line with standard intuition – it lowers consumption in the short run.

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1 Introduction

This paper studies the effects of a tax on energy use in a growth model where technological change and market structure are endogenous. Of particular interest is the interaction between changes in the inter-industry allocation of resources across manufacturing and energy production and the intra-industry effects within manufacturing. The latter are important because the manufacturing sector is the engine of growth of the economy.

There are several reasons why such an analysis is worthwhile. The following three stand out in light of recent events; others – like, e.g., reducing road congestion or reforming the structure of taxation to make it more efficient – are important but not as prominent in the current debate.

- The recent spike in the price of oil has once again focussed attention on how *energy prices* affect the economy in the short and the long run. The evidence is mixed. Hamilton (1988, 2003) argues that exogenous shocks to the price of oil explain most of the fluctuations of the US economy; Barsky and Killian (2002, 2004) and Killian (2006a, 2006b), in contrast, argue that they matter very little. It is fair to say, however, that the conventional wisdom is in line with Hamilton's view – that is, the price of oil drives business cycle fluctuations and long-run growth. A corollary to this view is the widespread belief (particularly in the US) that high standards of living require low energy prices.

- In industrialized countries the spike in the price of oil has also reignited the debate on how to reduce *dependence on foreign supply*. In light of the war in Iraq and the escalating tension with some large oil producers in the Arab world (e.g., Iran) and in Latin America (e.g., Venezuela), this argument acquires traction for geopolitical and national security reasons – especially the US. The question, of course, is how to accomplish a reduction of the dependence on foreign oil over a reasonable time-frame without harming the economy.

- Last but not least, there is the issue of *pollution and global warming*. Energy production and use is one of the main sources of harmful emissions, in general, and the main source of CO_2 emissions, in particular. CO_2, in turn, is the main factor in the building up of greenhouse gases that according to the growing consensus among natural scientists drive the rise in global temperature. The issue is far from settled, of course, and there is sharp disagreement between those who argue that global warming is man-made and those who argue that it is not, or that the projected costs of reducing it far outweigh the benefits; see, e.g., the heated debate following the recent release of the Stern Report by the UK government. Nevertheless, the notion that reducing pollution is desirable, and therefore that appropriate interventions in the energy sector are called for, is widely accepted.

In summary, there is ample motivation for studying the role of energy prices in the economy. In light of the stated goal of reducing the economy's energy intensity (the ratio of energy use to GDP) without inflicting undue harm, moreover, understanding the role of specific instruments like energy taxes becomes very important.

Over the last 10 years economists have placed more and more emphasis on the role of technological change in the analysis of energy, environmental, and climate policy.[1] The reason is that technology is now seen as a crucial factor in the assessment of the long-run costs and benefits of the proposed interventions. Perhaps surprisingly, however, the literature has not exploited to its full potential the modern theory of endogenous technological change to shed new light on these issues.[2] With this paper, I try to fill this gap.

I take a new look at the long-run implications of energy taxation through the lens of modern Schumpeterian growth theory. In particular, I use a model of the latest vintage that sterilizes the scale effect through a process of product proliferation that fragments the aggregate market into submarkets whose size does not increase with the size of the workforce.[3] The model is extremely tractable and yields a closed-form solution for the economy's transition path. This in turn allows me to study analytically the welfare effects of the energy tax.

My main finding is that under the plausible assumption that *energy demand is inelastic* there exists a hump-shaped relation between the energy

[1] This literature has grown so rapidly and extensively that any attempt at summarizing it here would do injustice to the many contributors. It is probably more productive to refer the reader to the recent reviews by Aghion and Howitt (1998, Ch. 5), Smulders (2000) and, in particular, Brock and Taylor (2005) and Xepapadeas (2005).

[2] One reason is that incorporating environmental externalities and resource scarcity increases dramatically the complexity of growth models. As a consequence, the early attempts have focussed mostly on first-generation models of endogenous innovation. A relatively small literature that developed recently has started to push the frontier harder and generate novel insights concerning the energy-growth relation. Two papers that deserve particular mention are Smulders and de Nooij (2003) and André and Smulders (2004). They build models that are close in spirit to what I do here. The main difference is that I use a model of endogenous innovation without the scale effect.

[3] Zeng and Zhang (2002) and Peretto (2003) have recently shown that these models have profound implications for the analysis of taxation.

tax and welfare. Interestingly, I obtain this shape abstracting from environmental externalities – a modeling choice that brings to the forefront how endogenous technological change alters dramatically the assessment of the short- and long-run *economic costs* of the energy tax.

The mechanism driving this result is the following. The tax on energy use changes relative after-tax input prices and induces manufacturing firms to substitute other inputs for energy in their production operations. As energy demand falls, the economy experiences a reallocation of resources from the energy sector to the manufacturing sector. This reallocation induces an increase of aggregate R&D, the sum of cost-reducing R&D internal to the firm and entrepreneurial R&D aimed at product variety expansion. Despite this reallocation, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. This follows from the fact that the increase in the size of the manufacturing sector attracts entry and, over time, the larger number of firms generates dispersion of R&D resources across firms and thus sterilizes the scale effect. Consequently, the growth rate of total factor productivity (TFP) in manufacturing is independent of the size of the manufacturing sector.

The core of this mechanism is the reallocation of resources from energy to manufacturing that generates a *temporary acceleration* of TFP growth. Under empirically plausible conditions there exists a range of tax rates such that this acceleration generates a ✓-shaped time profile of consumption whereby consumption drops on impact and then rises sufficiently fast and by a sufficient amount that welfare rises. In other words, the long-run gain due to endogenous technological change more than offsets the short-run pain – the fact that holding technology constant, the higher after-tax price of energy makes goods more expensive so that consumption falls.

I mentioned that the main body of the analysis abstracts from environmental externalities so that the capability of the tax to enhance welfare stems solely from its effect on the inter-sectoral allocation of resources. The intuition is that this reallocation mitigates some of the distortions – monopolistic pricing, firms' failure to internalize technological spillovers and other pecuniary externalities related to the interaction between incumbents and entrants – that characterize models of endogenous innovation. Hence, my positive analysis suggests that as a second-best instrument the energy tax has desirable effects independently of its role in addressing environmental problems. This feature of the analysis emphasizes how allowing for endogenous technological change alters drastically the assessment of the costs of policy interventions. In the final part of the paper I show how including environmental externalities enhances its potential to improve welfare through the familiar channel of pollution reduction.

The paper is organized as follows. In Section 2, I discuss the setup of the model. In Section 3, I construct the equilibrium of the market economy. In Section 4, I discuss the dynamic effects of the energy tax. In Section 5, I discuss pollution. I conclude in Section 6.

2 The model

2.1 Overview

I consider an economy populated by a representative household that supplies labor services inelastically in a competitive market. The household can also freely borrow and lend in a competitive market for financial assets.

Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or, in the case of entrants, set up operations. In addition to labor, production of consumption goods requires energy, which is supplied by a separate, competitive sector. Energy use and production generate pollution. The government taxes energy purchases and returns the proceeds in a lump-sum fashion to the household.[4]

The economy starts out with a given range of goods, each supplied by one firm. The household values variety and is willing to buy as many differentiated goods as possible. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses.

Once in the market, firms establish in-house R&D facilities to produce a stable flow of cost-reducing innovations. As each firm invests in R&D, it contributes to the pool of public knowledge and reduces the cost of future R&D. This allows growth at a constant rate in steady state, which is reached when the economy settles into a stable industrial structure.

2.2 Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-(\rho-\lambda)(s-t)} \log u(s) ds, \quad \rho > \lambda > 0 \tag{1}$$

[4] This ensures that the government balances the budget without introducing feedback effects that are not the focus of this paper.

subject to the flow budget constraint

$$\dot{A} = rA + WL + \tau E + \Pi_E - Y, \quad \tau \geq 0 \tag{2}$$

where ρ is the discount rate, λ is population growth, A is assets holding, r is the rate of return on financial assets, W is the wage rate, $L = L_0 e^{\lambda t}$, $L_0 \equiv 1$, is population size, which equals labor supply since there is no preference for leisure, and Y is consumption expenditure. In addition to asset and labor income, the household receives the lump-sum rebate of the energy tax revenues, τE, where τ is a per-unit tax and E is aggregate energy use. It also receives dividends Π_E from the energy sector.

The household has instantaneous preferences over a continuum of differentiated goods,[5]

$$\log u = \log \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3}$$

where ϵ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the mass of goods (the mass of firms) existing at time t.

The solution for the optimal expenditure plan is well known. The household saves if assets earn the reservation rate of return

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda \tag{4}$$

and taking as given this time-path of expenditure maximizes (3) subject to $Y = \int_0^N P_i C_i di$. This yields the demand schedule for product i,

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \tag{5}$$

With a continuum of goods, firms are atomistic and take the denominator of (5) as given; therefore, monopolistic competition prevails and firms face isoelastic demand curves.

[5] For simplicity I omit a term representing preference for environmental quality. The reason is that it is not necessary to develop my argument about energy taxes since I focus on the response of the decentralized market equilibrium and not on the socially optimal policy. I discuss the role of pollution externalities in Section 5.

2.3 Manufacturing: Production and Innovation

The typical firm produces one differentiated consumption good with the technology

$$X_i = Z_i^{\theta} \cdot F\left(L_{X_i} - \phi, E_i\right), \quad 0 < \theta < 1, \ \phi > 0 \tag{6}$$

where X_i is output,[6] L_{X_i} is production employment, ϕ is a fixed labor cost, E_i is energy use, and Z_i^{θ} is the firm's TFP, a function of the stock of firm-specific knowledge Z_i. The function $F(\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. Hence, (6) exhibits constant returns to rival inputs, labor and energy, and overall increasing returns. The associated total cost is

$$W\phi + C_X(W, P_E + \tau) Z_i^{-\theta} \cdot X_i, \tag{7}$$

where $C_X(\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments. The elasticity of unit cost reduction with respect to knowledge is the constant θ.

The firm accumulates knowledge according to the R&D technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{8}$$

where $\dot{Z}_i$ measures the flow of firm-specific knowledge generated by an R&D project employing L_{Z_i} units of labor for an interval of time dt and αK is the productivity of labor in R&D as determined by the exogenous parameter α and by the stock of public knowledge, K.

Public knowledge accumulates as a result of spillovers. When one firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, an R&D project that produces $\dot{Z}_i$ units of proprietary knowledge also generates $\dot{Z}_i$ units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

[6] For simplicity, I let X_i denote both the demand for and the production of good i.

which says that the technological frontier is determined by the average knowledge of all firms.[7]

The R&D technology (8), combined with public knowledge K, exhibits increasing returns to scale to knowledge and labor, and constant returns to scale to knowledge. This property makes constant, endogenous steady-state growth feasible.

2.4 The Energy Sector – and the rest of the world

Energy firms hire labor, L_E, to extract energy from natural resources (e.g. carbon, oil, gas), O. The energy-generation technology is $E = G(L_E, O)$, where $G(\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. The associated total cost is

$$C_E(W, P_O)\,E, \tag{9}$$

where $C_E(\cdot)$ is a standard unit-cost function homogeneous of degree one in the wage W and the price of resources P_O.

To fix ideas, I refer to resources as "oil" and assume that domestic supply is zero. In other words, I think of this as a small open economy that faces an infinitely elastic world supply. Corresponding to oil purchases, then, there is a flow of payments to the rest of the world. I show below that this flow is in units of labor (my numeraire, see below) so that the economy trades labor services for oil and the balanced trade conditions holds.

I ignore international assets flows and any other sort of interaction with the rest of the world, e.g., technological spillovers, import-export of goods other than the exchange of labor services for oil, and so on.[8]

This is the simplest way to model the energy sector for the purposes of this paper. Energy is produced with labor and natural resources purchased at a given price in the world market. The energy sector competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy. Energy purchases are taxed on a per-unit basis. This affects manufacturers' production costs, their demand for energy and labor, and the general equilibrium path of the economy.

[7] For a detailed discussion of the microfoundations of a spillovers function of this class, see Peretto and Smulders (2002).

[8] It is also possible to think of this as a small open economy that takes as given the world interest rate. Since the model has the property that the domestic interest rate jumps to its steady state level, given by the domestic discount rate, as long the small open economy has the same discount rate as the rest of the world the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.

3 Equilibrium of the Market Economy

This section constructs the symmetric Nash equilibrium of the manufacturing sector. It then characterizes the equilibrium of the energy sector. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the economy. The wage rate is the numeraire, i.e., $W \equiv 1$.

3.1 Equilibrium of the Manufacturing Sector

3.1.1 Incumbents

The typical manufacturing firm maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^\infty e^{-\int_t^s r(v)v} \Pi_{Xi}(s) ds.$$

Using the cost function (7), instantaneous profits are

$$\Pi_{Xi} = [P_i - C_X(1, P_E + \tau) Z_i^{-\theta}] X_i - \phi - L_{Z_i},$$

where L_{Z_i} is R&D expenditure. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technology (8), the demand schedule (5), $Z_i(t) > 0$ (the initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' innovation paths), and $\dot{Z}_j(t') \geq 0$ for $t' \geq t$ (innovation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, I follow Etro (2004) and assume that upon payment of a sunk cost $\beta P_i X_i$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.[9] Once in the market, the new firm implements price and R&D strategies that solve a problem identical to the one outlined above. Hence, entry yields value V_i. A free entry equilibrium, therefore, requires $V_i = \beta P_i X_i$.

The appendix shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$L_X = Y \frac{\epsilon - 1}{\epsilon} S_X^L + \phi N; \tag{10}$$

$$E = Y \frac{\epsilon - 1}{\epsilon} \frac{S_X^E}{P_E + \tau}, \tag{11}$$

[9] See Etro (2004) and Peretto and Connolly (2004) for a more detailed discussion of the microfoundations of this assumption.

where the shares of the firm's variable costs due to labor and energy are, respectively:

$$S_X^L \equiv \frac{WL_{X_i}}{C_X(1, P_E+\tau)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_E+\tau)}{\partial \log W};$$

$$S_X^E \equiv \frac{(P_E+\tau)\,E_i}{C_X(1, P_E+\tau)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_E+\tau)}{\partial \log (P_E+\tau)}.$$

Note that $S_X^L + S_X^E = 1$. Since the wage is normalized to 1, moreover, these shares are functions of the after-tax price of energy only.

Associated to these factor demands are the return to cost reduction and entry, respectively:

$$r = r_A \equiv \alpha \left[\frac{Y\theta(\epsilon-1)}{\epsilon N} - \frac{L_Z}{N}\right]; \tag{12}$$

$$r = r_N \equiv \frac{1}{\beta}\left[\frac{1}{\epsilon} - \frac{N}{Y}\left(\phi + \frac{L_Z}{N}\right)\right] + \hat{Y} - \hat{N}. \tag{13}$$

The dividend price ratio in (13) depends on the gross profit margin $\frac{1}{\epsilon}$. Anticipating one of the properties of the equilibria that I study below, note that in steady state the capital gain component of this rate of return, $\hat{Y} - \hat{N}$, is zero. Hence, the feasibility condition $\frac{1}{\epsilon} > r\beta$ must hold. This simply says that the firm expects to be able to repay the entry cost because it more than covers fixed operating and R&D costs.

3.2 Equilibrium of the Energy Sector

Given the cost function (9), competitive energy producers operate along the infinitely elastic supply curve

$$P_E = C_E\left(1, P_O\right). \tag{14}$$

In equilibrium, then, energy production is given by (11) evaluated at this pre-tax price. Defining the share of oil in energy costs as

$$S_E^O \equiv \frac{P_O O}{C_E\left(1, P_O\right) E} = \frac{\partial \log C_E(W, P_O)}{\partial \log P_O},$$

I can write the associated demands for labor and oil as:

$$L_E = E\frac{\partial C_E(W, P_O)}{\partial W} = Y\frac{\epsilon-1}{\epsilon}\frac{P_E S_X^E}{P_E+\tau}\left(1-S_E^O\right); \tag{15}$$

$$P_O O = E\frac{\partial C_E(W, P_O)}{\partial P_O} = Y\frac{\epsilon - 1}{\epsilon}\frac{P_E S_X^E}{P_E + \tau}S_E^O. \quad (16)$$

The share S_E^O depends only on the exogenous price of oil. Not surprisingly, (15) and (16) yield that the competitive energy producers make zero profits and thus pay zero dividends to the household. Consequently, I can set $\Pi_E = 0$ in (2).

3.3 General equilibrium

The model consists of the returns to saving, cost reduction and entry in (4), (12) and (13), the labor demands (10), (15), and the household's budget constraint (2).[10] Since the latter implies labor market clearing, equilibrium of the goods market follows from the fact that firms choose a point on their demand curves, and I have already imposed energy market equilibrium, to construct the general equilibrium of the economy, I now only need to look at the financial market.

Assets market equilibrium requires equalization of all rates of return (no-arbitrage), $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta Y$. Thus, the economy features a constant wealth to expenditure ratio. This property and log utility deliver a result that simplifies dramatically the analysis of dynamics. Substituting $A = \beta Y$ into (2), using the rate of return to saving in (4), and recalling that $\Pi_E = 0$, I obtain

$$0 = \beta(\rho - \lambda) + \frac{L + \tau E - Y}{Y},$$

which I can rewrite

$$\frac{Y}{L} = \frac{1}{1 - \beta(\rho - \lambda) - \tau\frac{E}{Y}} \equiv y^*, \quad (17)$$

where

$$\frac{E}{Y} = \frac{\epsilon - 1}{\epsilon}\frac{S_X^E}{P_E + \tau}.$$

[10] The household's budget constraint in (2) becomes the labor market clearing condition (see the appendix for the derivation):

$$L = L_N + L_X + L_Z + L_E + L_O,$$

where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and R&D operations of existing firms, L_E is aggregate employment in generation activity of energy firms and $L_O = P_O O$ is the balanced trade condition that states that the country exchanges labor services for oil.

This term depends only on parameters and the exogenous price of oil P_O. Since y^* is constant, then, the interest rate is $r=\rho$ at all times.

3.4 Dynamics

Because population grows, it is useful to work with the variable $n\equiv\frac{N}{L}$. The results just derived allow me to solve (8) and (12) for

$$\hat{Z}=\alpha\frac{L_Z}{N}=\frac{y^*}{n}\frac{\alpha\theta\left(\epsilon-1\right)}{\epsilon}-\rho. \tag{18}$$

An important feature of this equation is that $\frac{L_Z}{N}=0$ for

$$n\geq\bar{n}\equiv y^*\frac{\alpha\theta\left(\epsilon-1\right)}{\rho\epsilon}.$$

This is the familiar effect of the number of firms on the individual firm's market share and thus on the incentives to do R&D.[11]

I now substitute these results into (13) to obtain

$$\hat{n}=\begin{cases}\frac{1}{\beta}\left[\frac{1-\theta(\epsilon-1)}{\epsilon}-\left(\phi-\frac{\rho}{\alpha}\right)\frac{n}{y^*}\right]-\rho & n<\bar{n}\\ \frac{1}{\beta}\left[\frac{1}{\epsilon}-\phi\frac{n}{y^*}\right]-\rho & n\geq\bar{n}\end{cases}.$$

The general equilibrium of the model thus reduces to a single differential equation in the mass of firms per capita. Figure 1 illustrates dynamics.[12] If $\alpha\phi>\rho$, the entry rate is always falling. In contrast, if $\alpha\phi\leq\rho$ the entry rate is initially rising or constant, until the economy crosses the threshold $\bar{n}$ when the entry rate starts falling. In all cases, the economy converges to

$$n^*=\begin{cases}\frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi-\frac{\rho}{\alpha}}y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi\alpha-\rho}<\frac{\theta(\epsilon-1)}{\rho\epsilon}\\ \frac{\frac{1}{\epsilon}-\rho\beta}{\phi}y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi\alpha-\rho}\geq\frac{\theta(\epsilon-1)}{\rho\epsilon}\end{cases}. \tag{19}$$

These dynamics make clear that $\phi>0$ kills the possibility of endogenous growth through product proliferation because the term ϕN on the right hand side of the resources constraint implies that the equation cannot hold for a given labor endowment if N grows too large.[13]

[11] See Peretto (1998, 1999) for a discussion of this property in this class of models.

[12] For simplicity I ignore the non-negativity constraint on $\dot{N}$. I can do so without loss of generaility because population growth implies that the mass of firms grows all the time. See Peretto (1998) for a discussion of this property.

[13] See Peretto and Connolly (2004) for a detailed discussion of this property in Schumpeterian models of endogenous growth.

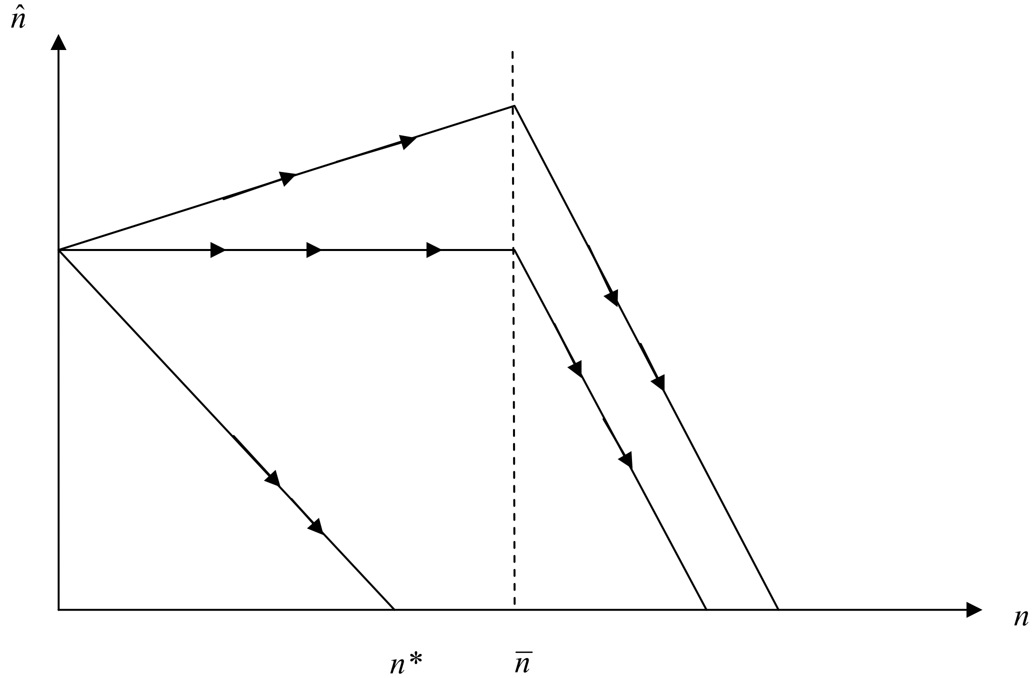

Figure 1: General Equilibrium Dynamics

The solutions in (19) exist only if the feasibility constraint $\frac{1}{\epsilon} > \rho\beta$ holds. The interior steady state with both vertical and horizontal R&D requires the more stringent conditions $\alpha\phi > \rho$ and

$$\rho\beta + \frac{\theta(\epsilon - 1)}{\epsilon} < \frac{1}{\epsilon} < \rho\beta + \frac{\alpha\phi}{\rho}\frac{\theta(\epsilon - 1)}{\epsilon}.$$

It then yields

$$\frac{y^*}{n^*} = \frac{\phi - \frac{\rho}{\alpha}}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta} \tag{20}$$

so that

$$\hat{Z}^* = \frac{\phi\alpha - \rho}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta}\frac{\theta(\epsilon - 1)}{\epsilon} - \rho. \tag{21}$$

Notice how the steady-state growth rate of productivity in manufacturing is independent of conditions in the energy market because the sterilization of the scale effect implies that it does not depend on the size of the manufacturing sector and therefore on the inter-sectoral allocation of labor.

To perform experiments, I shall focus on this region of parameter space and work with the equation

$$\hat{n} = \nu - \left(\phi - \frac{\rho}{\alpha}\right)\frac{n}{\beta y^*}, \quad \nu \equiv \frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - \rho.$$

This is a logistic equation (see, e.g., Banks 1994) with growth coefficient ν and crowding coefficient $\left(\phi - \frac{\rho}{\alpha}\right)\frac{1}{\beta y^*}$. Using the value n^* in (19) – also called carrying capacity – I can rewrite it as

$$\hat{n} = \nu\left(1 - \frac{n}{n^*}\right), \tag{22}$$

which has solution

$$n(t) = \frac{n^*}{1 + e^{-\nu t}\left(\frac{n^*}{n_0} - 1\right)}, \tag{23}$$

where n_0 is the initial condition.

4 The effects of the energy tax

In this section I analyze the effects of the energy tax. I begin with a discussion of the conditions under which it raises consumption expenditure so that the market for manufacturing goods expands. Next, I show how the reallocation of resources associated to that expansion affects the path of TFP, the CPI and therefore of (real) consumption per capita. Finally, I show that under plausible conditions welfare is a hump-shaped function of the tax.

4.1 Expenditure

The first step in the evaluation of the effects of the energy tax on growth and welfare is to assess its effect on expenditure. The following property of the energy demand function (11) is quite useful.

Lemma 1 *Let*

$$\epsilon_X^E \equiv -\frac{\partial \log E}{\partial \log (P_E + \tau)}.$$

Then, $\epsilon_X^E \leq 1$ *if*

$$\frac{\partial S_X^E}{\partial (P_E + \tau)} \geq 0,$$

which is true if

$$\frac{\partial L_X}{\partial (P_E + \tau)} \leq 0,$$

that is, if labor and energy are gross complements in (6).

Proof. See the Appendix. ■

In words, this says that energy demand is inelastic, i.e., $\epsilon_X^E \leq 1$, when the energy share of manufacturing cost, S_X^E, is non-decreasing in the after-tax price of energy. Energy demand, conversely, is elastic when the the energy cost share in manufacturing is decreasing in $P_E + \tau$. The effect of the after-tax price of energy on the energy cost share, in turn, depends on whether labor and energy are gross complements or gross substitutes. I now use this result to derive one of the key ingredients for the analysis of the growth and welfare effects of the tax.

Proposition 2 *Assume that the production technology (6) exhibits gross complementarity between labor and energy so that energy demand is inelastic,* $\epsilon_X^E \leq 1$*. Then,* $y^*(\tau)$ *is a monotonically increasing function with domain* $\tau \in [0, \infty)$ *and codomain* $[y^*(0), y^*(\infty))$*, where:*

$$y^*(0) = \frac{1}{1 - \beta(\rho - \lambda)};$$

$$y^*(\infty) = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon}}.$$

Assume, in contrast, that labor and energy are gross substitutes so that $\epsilon_X^E > 1$*. Then,* $y^*(\tau)$ *is a hump-shaped function of* τ *with the same domain as before and codomain and codomain* $[y^*(0), y^*(\infty))$*, where:*

$$y^*(0) = y^*(\infty) = \frac{1}{1 - \beta(\rho - \lambda)}.$$

Proof. See the Appendix. ■

The mechanism driving this result highlights the importance of the substitution possibilities between labor and energy in manufacturing. If labor and energy are gross complements, higher manufacturing employment requires higher energy use, and this dampens the negative effect of the increase in the after-tax price of energy on energy demand. If, in contrast, labor and energy are gross substitutes, higher manufacturing employment requires lower energy use and thereby amplifies the negative effect of the after-tax price increase on energy demand.

To see this property in sharper detail, it is useful to consider the following example.

Example 3 *Consider a CES production function of the form*

$$X_i = Z_i^{\theta}\left[(L_{X_i} - \phi)^{\sigma} + E_i^{\sigma}\right]^{\frac{1}{\sigma}}, \quad \sigma \leq 1$$

As is well known, this contains as special cases the linear production function ($\sigma = 1$) *wherein inputs are perfect substitutes, the Cobb-Douglas* ($\sigma = 0$) *wherein the elasticity of substitution between inputs is equal to* 1*, and the Leontief* ($\sigma = -\infty$) *wherein inputs are perfect complements. The associated unit-cost function is*

$$C_{X_i} = Z_i^{-\theta}\left[W^{\frac{\sigma}{\sigma-1}} + (P_E + \tau)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}.$$

From this one derives (recall that $W \equiv 1$*):*

$$\begin{aligned} S_X^E &= \frac{1}{1 + (P_E + \tau)^{\frac{\sigma}{1-\sigma}}}; \\ \epsilon_X^E &= 1 + \frac{\sigma}{1-\sigma}\frac{1}{1 + (P_E + \tau)^{-\frac{\sigma}{1-\sigma}}} = 1 + \frac{\sigma}{1-\sigma}\left(1 - S_X^E\right). \end{aligned}$$

Hence, $\sigma \leq 0$ *yields* $\epsilon_X^E \leq 1$ *and* $\frac{dy^*}{d\tau} > 0$ *for all* τ*. In contrast,* $0 < \sigma \leq 1$ *yields* $\epsilon_X^E > 1$ *and* $\frac{dy^*}{d\tau} > 0$ *for* $\tau < \bar{\tau}$ *where*

$$\bar{\tau} \equiv \arg\text{solve}\left\{1 = \frac{\tau\epsilon_X^E}{P_E + \tau}\right\}.$$

The main message of Proposition 1 is that expenditure rises with the tax when energy demand is inelastic because the induced fall in energy use is not so dramatic that total tax revenues fall. In other words, the economy operates on the upward sloping part of the energy tax revenue curve.

4.2 The CPI: how the cost of energy and TFP affect consumption

Consider now the effects on consumption. The price index of a basket of consumption goods – the CPI of this economy – is

$$P_Y = \left[\int_0^N P_j^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}.$$

Accordingly, the price strategy (32) yields *real* expenditure per capita as

$$\frac{y^*}{P_Y} = \frac{\epsilon - 1}{\epsilon} \frac{y^*}{c^*} N^{\frac{1}{\epsilon-1}} Z^\theta,$$

where to simplify the notation I define $c^* \equiv C_X(1, C_E(1, P_O) + \tau)$. This unit cost is pinned down by exogenous parameters.

Why look at real expenditure? Because it measures the flow of consumption in the utility function (3) and thus is relevant for welfare. Moreover, one can reinterpret (3) as a production function for a final homogenous good assembled from intermediate goods and define aggregate TFP as

$$T = N^{\frac{1}{\epsilon-1}} Z^\theta. \tag{24}$$

Accordingly,

$$\hat{T}(t) = \frac{1}{\epsilon - 1} \hat{N}(t) + \theta \hat{Z}(t).$$

In steady state this gives

$$\hat{T}^* = \frac{\lambda}{\epsilon - 1} + \theta \hat{Z}^* \equiv g^*, \tag{25}$$

where $\hat{Z}^*$ is given by (21). Observe how g^* is independent of conditions in the energy market and of population size and growth.

A nice feature of this model is that I can compute TFP in closed form along the transition path. To bring this feature to the forefront, notice that according to (20) in steady state

$$\frac{y^*}{n^*} = \frac{y_0}{n_0} \Rightarrow \frac{y^*}{y_0} = \frac{n^*}{n_0}.$$

Now define

$$\Delta \equiv \frac{n^*}{n_0} - 1 = \frac{y^*}{y_0} - 1.$$

This is the percentage increase in expenditure that the economy experiences in response to changes in fundamentals and/or policy parameters. It fully summarizes the effects of such changes on the scale of economic activity. The following proposition characterizes how changes in scale affect the manufacturing sector.

Proposition 4 *At any time $t > 0$ the log of TFP is*

$$\begin{aligned} \log T(t) &= \log\left(Z_0^{\theta} n_0^{\frac{1}{\epsilon-1}}\right) + g^* t \\ &\quad + \frac{\gamma\Delta}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}, \end{aligned} \tag{26}$$

where

$$\gamma \equiv \theta\frac{\alpha\theta(\epsilon-1)}{\epsilon}\frac{y^*}{n^*} = \theta\frac{\theta(\epsilon-1)}{\epsilon}\frac{\phi\alpha - \rho}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta}.$$

Moreover,

$$\frac{d\log T(t)}{d\tau} = \left[\frac{\gamma}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon-1}\frac{1}{1+\Delta}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}\right]\frac{d\Delta}{d\tau} > 0.$$

Proof. See the Appendix. ■

The TFP operator in (26) has two transitional components. The first is the cumulated gain from cost reduction, $Z(t)$, the second is the cumulated gain from product variety per capita, $n(t)$. The mechanism driving these components is quite intuitive: a tax on energy use changes relative after-tax input prices and induces manufacturing firms to substitute labor for energy in their production operations. This standard effect is associated to an increase of aggregate R&D employment, the sum of cost-reducing R&D internal to the firm and entrepreneurial R&D aimed at product variety expansion. Despite this reallocation, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. Average R&D, in other words, does not increase. This follows from the fact that the increase in the size of the manufacturing sector, measured by the rise in aggregate expenditure on consumption goods, raises the returns to entry. Over time the larger number of firms generates dispersion of R&D resources and sterilizes the scale effect. Consequently, the growth rate is independent of the size of the manufacturing sector.

Summarizing, the energy tax reallocates labor from the energy sector to the manufacturing sector. The increase in the size of the manufacturing sector generates a *temporary* growth acceleration.

If one ignores these effects of endogenous technological change – if one posits that T grows at an exogenous rate – consumption depends on the tax only through y^*/c^*. The following proposition characterizes this case.

Proposition 5 *Assume that technology does not adjust in response to the energy tax. Then,*

$$\frac{d}{d\tau}\left(\log\frac{y^*}{c^*}\right) < 0 \textit{ for all } \tau.$$

Proof. See the Appendix. ■

This is an important result. It says that endogenous technological change is *necessary* to obtain welfare gains from the energy tax. The reason is that, in line with intuition, the energy tax raises the cost of production of goods so that the CPI rises and consumption falls.

4.3 Welfare

I now investigate how the long-run effects of endogenous technological change offset the short-run cost (if any) of the energy tax. As I mentioned, to emphasize the importance of endogenous technology, I ignore for now environmental quality in the household's preferences so that the welfare gains of the tax stem solely from the fact that the reallocation of resources from energy to manufacturing accelerates temporarily the pace of endogenous technological change and yields a long-run TFP gain.

Proposition 6 *Let* $\log u^*(t)$ *and* U^* *be, respectively, the instantaneous utility index (3) and welfare function (1) evaluated at* y^**. Then, a path starting at time* $t = 0$ *is characterized by:*

$$\begin{aligned}\log u^*(t) &= \log\frac{y^*}{c^*} + g^*t \\ &\quad + \frac{\gamma\Delta}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}};\end{aligned} \tag{27}$$

$$
\begin{aligned}
U^* &= \frac{1}{\rho-\lambda}\left[\log\frac{y^*}{c^*}+\frac{g^*}{\rho-\lambda}+\frac{\gamma\Delta}{\rho-\lambda+\nu}\right] \\
&\quad +\frac{1}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}dt.
\end{aligned} \tag{28}
$$

Proof. See the Appendix. ■

The first term in (28) captures the role of steady-state real expenditure calculated holding technology, T, constant; the second captures the role of steady state growth of income per capita, g^*; the third is the contribution from the gain in firm-level productivity, Z, due to the transitional acceleration of firm-level cost-reduction; the fourth term captures the contribution of the gain in n due to the transitional acceleration of product variety expansion. Figure 2 illustrates the underlying path of the log of consumption. Since the CPI jumps up on impact, there is an instantaneous drop in consumption, followed by faster than trend growth, with eventual convergence to a higher steady-state growth path, parallel to the initial one. Thus, all the gains from the energy tax stem from *level effects* spread over time. This is where the model's property that the transition path has an analytical solution comes very handy since it allows one to see exactly how the intertemporal trade-off plays out.

I now show that under plausible conditions endogenous technological change yields that welfare is a hump-shaped function of τ, as in Figure 3.

Proposition 7 *Assume that the production technology (6) exhibits gross complementarity between labor and energy so that energy demand is inelastic, $\epsilon_X^E \leq 1$. Consider an economy on the equilibrium path induced by the introduction of an energy tax $\tau > 0$, starting from the equilibrium with no energy tax, i.e., $\tau = 0$. Then, U^* is a hump-shaped function of τ iff*

$$
\frac{\gamma+\frac{\nu}{\epsilon-1}}{\rho-\lambda+\nu} > \frac{\epsilon}{\epsilon-1}\left[\frac{1}{\epsilon}-\beta\left(\rho-\lambda\right)\right]. \tag{29}
$$

Otherwise, U^* is decreasing in τ for all $\tau \geq 0$. A sufficient condition for inequality (29) to hold is

$$
\frac{g^*}{\rho}+\theta > \frac{1}{\epsilon-1}.
$$

Proof. See the Appendix. ■

The second inequality in this proposition provides a *sufficient* condition for existence of an "optimal" energy tax $\tau^* > 0$. To see whether this

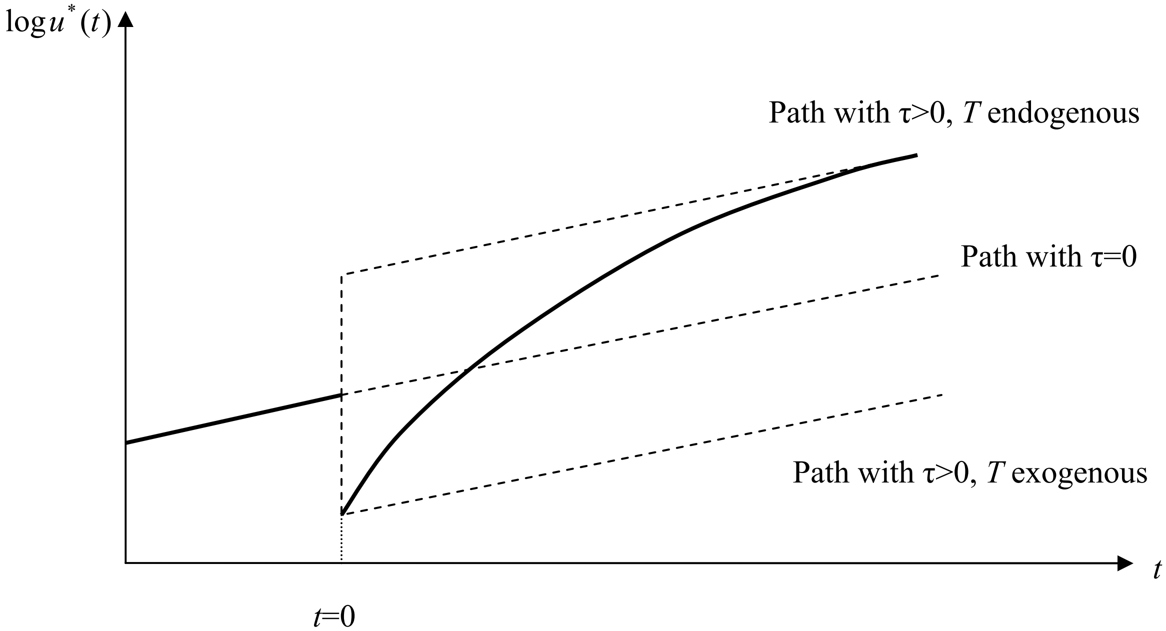

Figure 2: The path of consumption

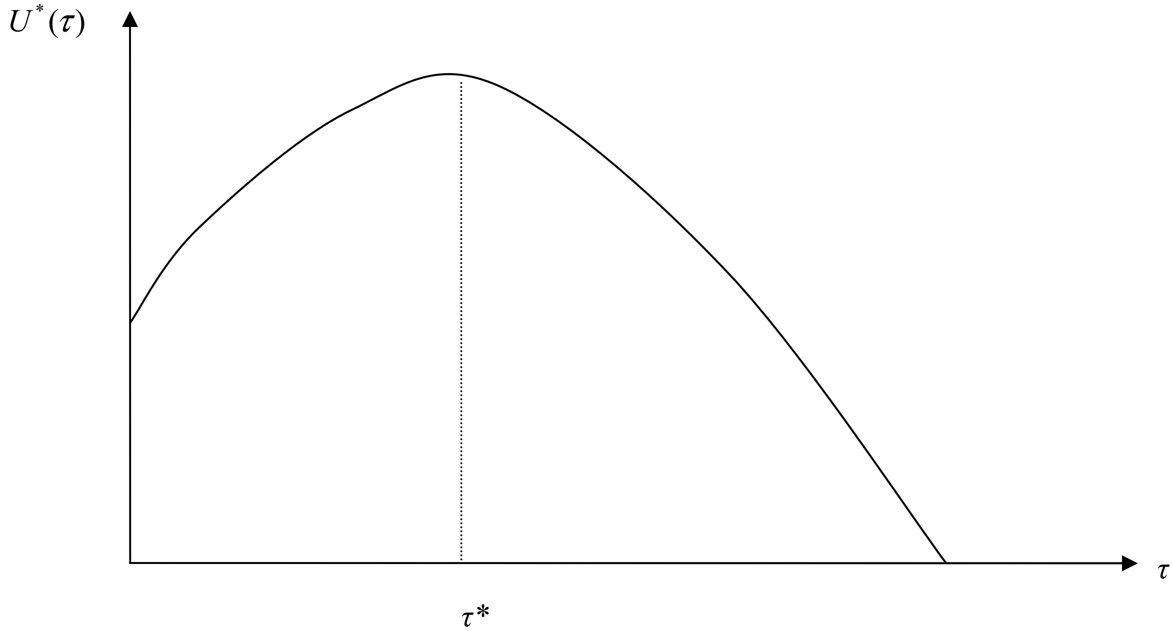

Figure 3: The energy tax and welfare

condition is likely to hold in reality, recall that $\rho = r^*$ and that $\frac{1}{\epsilon}$ is the typical firm's profit margin. Then, one can compute from the data for the US economy (see, e.g., Barro and Sala-i-Martin 2004)

$$\frac{g^*}{r^*} = \frac{.02}{.04} = \frac{1}{2}.$$

The condition then holds for all $\theta > 0$ if

$$\frac{1}{2} > \frac{1}{\epsilon - 1} \Longrightarrow \epsilon > 3 \Longrightarrow \frac{1}{\epsilon} < \frac{1}{3}.$$

In other words, the sufficient condition holds in the US economy given that typically we observe average profit margins of less than 33%, corresponding to estimates of ϵ that are larger than 3. Notice, moreover, that the sufficient condition in the proposition is in fact less restrictive than this, because what I need is

$$\frac{g^*}{r^*} + \theta > \frac{1}{\epsilon - 1},$$

so that I need the estimates of ϵ to satisfy

$$\epsilon > 1 + \frac{1}{\frac{g^*}{r^*} + \theta} = 1 + \frac{1}{\frac{1}{2} + \theta}.$$

Unfortunately, I do not have independent estimates of θ, the elasticity of cost reduction. However, if I fix it at $\frac{1}{2}$, then I need $\epsilon > 2$. Notice that $\epsilon = 2$ yields a profit margin of 50%. If I set θ at $\frac{3}{4}$, then I need $\epsilon > 1.8$.

5 Pollution

The previous section has shown that there exists an "optimal" tax abstracting from the benefits of pollution reduction. This section investigates the role of pollution, a by-product of economic activity.

Assume that labor use does not pollute while energy use does.[14] Denote emissions per firm with q_i. Positing a linear relation, $q_i = E_i$, to simplify

[14] If the damage that the firm inflicts on the environment is a function of the level of output, $q_i = X_i$, environmental quality falls without bound since output grows forever. This is not a useful assumption if one does not specify an abatement technology that allows a growing economy to invest resources and reduce pollution. (For an example of work that follows this approach, see Peretto 2006a.) This is particularly important if there is a minimum level of environmental quality below which economic activity itself is not sustainable. In the context of this model a more reasonable proxy for emissions per firm is energy use since pollution does not rise with output if output growth comes from TFP as opposed to more input use. It is the fact that energy use ultimately involves burning of fossil fuels that makes it so important for pollution. Higher ouptut at unchanged use of energy inputs should not *per se* contribute to emissions.

the algebra, I can write pollution damages as

$$D = \int_0^N q_i di = \int_0^N E_i di = E. \tag{30}$$

In this formulation pollution depends only on aggregate energy use. Allowing for dependence on production of energy as well (which in equilibrium equals use) does not add insight to the analysis.

To see how pollution affects utility, I augment (3) as follows

$$\log u = \log \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \xi \log (1 + D), \quad \epsilon > 1, \ \xi > 0 \tag{31}$$

where as before ϵ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the mass of goods existing at time t. The new parameter ξ is the elasticity of utility with respect to pollution. $D = 0$ corresponds to a pristine environment; $D > 0$ produces a "bad" that reduces the flow of utility.

Recall that in equilibrium (11) yields

$$E = L \frac{y^* (\epsilon - 1)}{\epsilon (P_E + \tau)} S_X^E.$$

This expression and (30) say that pollution damages, $D = E$, grow over time because of population growth. Therefore, this economy exhibits the property that the *extensive* component of growth of income per capita – product variety expansion, which is tied to population growth – has a dirty side, while the *intensive* one – cost reduction, which is not related to population growth – does not. The reason is straightforward. Intensive growth is driven by Hicks neutral factor augmenting technological change that does not require an expanding resource base. In contrast, extensive growth is tied to the resource base – the labor endowment – because the expansion of the variety of products implies replication of fixed labor costs. In fact, population growth *drives* product variety expansion. This semi-endogenous component of the rate of growth of income per capita has the implication that in the presence of infinitely elastic supply of oil it also drives energy use and thus pollution.[15]

It is desirable to take action to offset the dirty side of economic growth. Suppose, for example, that the government allocates a fraction $1 - f$ of the

[15] If oil supply is finite, population growth drives explosive growth of the price of oil; see Peretto (2006c) for an analysis of this case.

energy tax revenues to abatement activities that clean up emissions. Then it is possible to construct an equilibrium where $D = E - (1 - f)\,\tau E = 0$ so that the loss of utility from pollution is eliminated, i.e., $-\xi \log D = 0$. This requires setting the lump-sum rebate of tax revenues to the household at

$$f\tau E = (\tau - 1)\,E.$$

Accordingly, (17) becomes

$$y^* = \frac{1}{1 - \beta\,(\rho - \lambda) + (\tau - 1)\,\frac{\epsilon-1}{\epsilon}\frac{S_X^E}{P_E+\tau}}.$$

Now observe that this policy eliminates pollution damages from the welfare function and leaves the remainder of the analysis virtually identical to that of the previous section. The only difference is that now there is the additional constraint that the tax must generate sufficient revenues to pay for fully cleaning up the environment. This is why in the expression above the term $\tau - 1$ appears in place of τ. It is then straightforward to replicate the previous analysis and show that welfare is again hump-shaped in τ.

This type of intervention corresponds to the "Kindergarten Rule" discussed by Brock and Taylor (2003, 2005). Notice that this is not a first-best policy since I am not solving a social planning problem taking into account all distortions. The point of this example is simply to show that introducing pollution externalities does not change the substance of the analysis of the previous section.

6 Conclusion and suggestions for further research

In this paper I studied the effects of a tax on energy use in a growth model where technological change and market structure are endogenous. I focussed in particular on the interaction between changes in the inter-industry allocation of resources across the manufacturing and energy sectors and the intra-industry effects within manufacturing. I found that under the plausible assumption that *energy demand is inelastic* there exists a hump-shaped relation between the tax and welfare.

The mechanism driving this result is the following. The tax induces manufacturing firms to substitute labor for energy in their production operations. As energy demand falls, the economy experiences a reallocation of labor from the energy sector to the manufacturing sector. This reallocation induces an increase of aggregate R&D employment. Despite this increase, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. Average R&D, in other words, does not increase.

The core of the mechanism, thus, is that the energy tax reallocates labor from energy to manufacturing and thereby generates a *temporary acceleration* of TFP growth that in the long run can offset the short-run pain of the tax due to the fact that, holding technology constant, the higher after-tax price of energy makes manufacturing goods more expensive. If the economy has growth-favoring fundamentals and patient households, there is a range of tax rates such that the lower prices at the end of the transition drive consumption up to the point where it dominates the intertemporal trade-off and welfare rises.

Skeptics of environmental taxes usually point out that results like these depend crucially on the willingness and capability of the government not to divert revenues to wasteful uses. This is a legitimate point. However, it is an argument concerning the proper functioning of the government in general, not an objection to environmental taxes per se. Waste is bad regardless of the particular tax instrument that funds it. Moreover, my analysis does not require the government to make a particularly enlightened use of tax revenues; it simply posits that it rebates them to the households in a lump-sum fashion. Whether this implies an unrealistic belief in the proper functioning of the political process is beyond the scope of this paper's analysis.

As a robustness check on my story, one can investigate the following extensions of the model discussed in this paper. (Most of these extension are currently work in progress or on my agenda for the near future.)

- The most obvious extension is endogenous labor supply. For example, I can easily show that with log utility defined over consumption and leisure the qualitative results are exactly the same. However, the interesting aspect of this extension is that the results can in fact be much stronger. If instead of rebating the energy tax revenues in a lump-sum fashion, the government uses them to pay down, say, a distortionary tax on labor income, then there is an additional *revenue-recycling effect* that expands labor supply, aggregate employment, and thereby boosts the growth acceleration driving my welfare result.

- As shown in Section 5, population growth generates escalating pollution. This requires targeted interventions. I discussed one that fully eliminates pollution under the assumption that pollution is a flow. The analysis extends easily to the case of stock pollution with no substantial change in the main insight.

- Population growth also puts pressure on the resource endowment and thus generates escalating rents. I bypassed this complication by using the small open economy assumption, which in practice turns a finite supply of oil into an infinitely elastic one. This convenient simplification might raise doubts about the robustness of my result. It turns out, however, that the result carries over to the case of scarcity due either to the fact that the economy is closed and has a finite endowment of oil (Peretto 2006c) or to the fact that the economy is not small and thus affects the world price of oil. This is important because it implies that in general the result is robust to plausible extensions that allow for a positive relation between (domestic) oil demand and the price at which (domestic) energy firms purchase oil.

- It is worth emphasizing that at issue here is the behavior of resource prices, not whether the economy is capable of long-run growth. Escalating resource prices are consistent with long-run growth in the presence of scarcity. In fact, escalating prices is precisely how the economy copes with resource scarcity. My assumption that TFP in manufacturing is Hicks neutral implies that growth of income per capita is feasible at *unchanged physical inputs use*, and thereby makes quite starkly the point that the scarcity signaled by escalating resource prices is due to population growth, not to growth of income per capita.

- An important aspect of scarcity is that it requires resource-augmenting technical change in the energy sector in order to offset the upward pressure on prices due to population growth. There is then an additional trade-off because resource-augmenting R&D in the energy sector competes for resources with R&D in manufacturing. On the other hand, this gives cumulative effects along the vertical production structure so that cost reductions in energy production ultimately show up in the prices of consumption goods. In my analysis I abstracted from this issue. I expect the basic result not to change, but I need to work out this case carefully to check.

- A related topic is costly abatement undertaken by firms. In this case, the scarcity that technological change must overcome is that of the environment as a sink for waste as opposed to as a source for oil and other fossil fuels. This entails a trade-off between allocating resources to emissions abatement as opposed to cost-reduction and product-creation. I analyzed this trade-off in a model of this class in Peretto (2006a), where I looked at first-best emission taxes. This trade-off is absent from the analysis undertaken here. In future work, I plan to extend the model to allow for this important dimension of the problem and study how energy taxes affect welfare when both the rate and direction of technological change can respond. The preliminary work that I have done so far suggests that the insight presented here carries over to the more general setup.

- It is possible to extend the model to the case of an economy with a non-zero endowment of oil, and look at how a broad oil tax brings about a reduction of dependence on foreign sources (see Peretto 2006b). This reduction can be envisioned as a smooth transition were eventually imports go to zero and the economy relies only on domestic sources.

7 Appendix

7.1 The typical firm's behavior

To characterize the typical firm's behavior, consider the Current Value Hamiltonian

$$CVH_i = [P_i - C_X(1, P_E + \tau)Z_i^{-\theta}]X_i - \phi - L_{Z_i} + z_i \alpha K L_{Z_i},$$

where the costate variable, z_i, is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable; R&D investment, L_{Z_i}, and the product's price, P_i, are the control variables. Firms take the public knowledge stock, K, as given.

Since the Hamiltonian is linear, one has three cases. The case $1 > z_i \alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest. The case $1 < z_i \alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in R&D, this case violates the general equilibrium conditions and is ruled out. The first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $1 = z_i \alpha K$, the constraint on the state variable, (8), the terminal condition,

$$\lim_{s \to \infty} e^{-\int_t^s r(v)v} z_i(s) Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r = \frac{\dot{z}_i}{z_i} + \theta C_X(1, P_E + \tau) Z_i^{-\theta-1} \frac{X_i}{z_i},$$

that defines the rate of return to R&D as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(1, P_E + \tau) Z_i^{-\theta} \frac{\epsilon}{\epsilon - 1}. \tag{32}$$

Peretto (1998, Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $1 = z_i \alpha K$ holds, and equilibrium is symmetric.

The cost function (7) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_E + \tau)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$E_i = \frac{\partial C_X(W, P_E + \tau)}{\partial (P_E + \tau)} Z_i^{-\theta} X_i.$$

Then, the price strategy (32), symmetry and aggregation across firms yield (10) and (11).

Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z / N$, where L_Z is aggregate R&D. Taking logs and time derivatives of $1 = z_i \alpha K$ and using the demand curve (5), the R&D technology (8) and the price strategy (32), one reduces the first-order conditions to (12).

Taking logs and time-derivatives of V_i yields

$$r = \frac{\Pi_{Xi}}{V_i} + \frac{\dot{V}_i}{V_i},$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium of the capital market. It requires that the rate of return to firm ownership equal the rate of return to a loan of size V_i. The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation).

In symmetric equilibrium the demand curve (5) yields that the cost of entry is $\beta \frac{Y}{N}$. The corresponding dmand for labor in entry is $L_N = \dot{N} \beta \frac{Y}{N}$. The case $V > \beta \frac{Y}{N}$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta \frac{Y}{N}$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $\dot{N} = 0$. Free-entry requires $V = \beta \frac{Y}{N}$. Using the price strategy (32), the rate of return to entry becomes (13).

7.2 The economy's resources constraint and balanced trade

I now show that the household's budget constraint reduces to the economy's labor market clearing condition, and that this condition contains the balanced trade condition stating that the economy trades labor services for oil. Starting from (2), recall that $A = NV$ and $rV = \Pi_X + \dot{V}$. Substituting into (2) yields

$$\dot{N}V = N\Pi_X + L + \tau E + \Pi_E - Y.$$

Observing that $N\Pi_X = NPX - L_X - L_Z - (P_E + \tau) E$, $NPX = Y$ and $\Pi_E = P_E E - L_E - P_o O$, this becomes

$$L = \dot{N}V + L_X + L_Z + L_E + P_o O.$$

Now recall that the free entry condition yields that total employment in entrepreneurial activity is $L_N = \dot{N}V$. Finally, let $L_O = P_O O$ denote the amount of labor exchanged for oil, and substitute this balanced trade condition into the expression above to write

$$L = L_N + L_X + L_Z + L_E + L_o.$$

This says that the small open economy allocates labor to five activities: creation of new goods/firms, production of goods and reduction of production costs for existing firms, generation of energy and "extraction" of oil. Extraction is in quotation marks because it takes the form of an exchange of the domestic resource (labor) for the foreign one (oil). In other words, trade is the extraction technology available to a resource poor economy.

7.3 Proof of Lemma 1

Observe that

$$\epsilon_X^E \equiv -\frac{\partial \log E}{\partial \log (P_E + \tau)} = 1 - \frac{\partial \log S_X^E}{\partial \log (P_E + \tau)} = 1 - \frac{\partial S_X^E}{\partial (P_E + \tau)} \frac{P_E + \tau}{S_X^E}$$

so that $\epsilon_X^E \leq 1$ if

$$\frac{\partial S_X^E}{\partial (P_E + \tau)} = \frac{\partial}{\partial (P_E + \tau)} \left(\frac{(P_E + \tau) E}{(P_E + \tau) E + L_X} \right) \geq 0.$$

This in turn is true if

$$\begin{aligned} \left(1 - S_X^E\right) \frac{\partial ((P_E + \tau) E)}{\partial (P_E + \tau)} - S_X^E \frac{\partial L_X}{\partial (P_E + \tau)} &\geq 0 \\ \left(1 - S_X^E\right) \left[\frac{\partial ((P_E + \tau) E)}{\partial (P_E + \tau)} + \frac{\partial L_X}{\partial (P_E + \tau)} \right] - \frac{\partial L_X}{\partial (P_E + \tau)} &\geq 0. \end{aligned}$$

Recall now that total cost is increasing in $P_E + \tau$ so that

$$\frac{\partial\left(\left(P_E+\tau\right)E\right)}{\partial\left(P_E+\tau\right)}+\frac{\partial L_X}{\partial\left(P_E+\tau\right)}>0.$$

It follows that

$$\frac{\partial L_X}{\partial\left(P_E+\tau\right)}\leq 0$$

is a sufficient condition for $\epsilon_X^E \leq 1$ since it implies that both terms in the inequality above are positive.

7.4 Proof of Proposition 2

Use (11) and the fact that $Y = Ly^*$ to rewrite (17) as

$$y^*=\frac{1+\tau\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}. \tag{33}$$

Then,

$$\begin{aligned}
\frac{dy^*}{d\tau} &= \frac{1}{1-\beta\left(\rho-\lambda\right)}\frac{d\left(\tau\frac{E}{L}\right)}{d\tau} \\
&= \frac{\frac{1}{L}}{1-\beta\left(\rho-\lambda\right)}\left[E+\tau\frac{\partial E}{\partial\tau}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1+\tau\frac{\partial E}{\partial\tau}\frac{1}{E}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1+\frac{\tau}{P_E+\tau}\frac{\partial\log E}{\partial\log\left(P_E+\tau\right)}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1-\frac{\tau}{P_E+\tau}\epsilon_X^E\right].
\end{aligned}$$

This expression is positive if $\epsilon_X^E \leq 1$. If, in contrast, $\epsilon_X^E > 1$ this expression changes sign at

$$1-\frac{\tau}{P_E+\tau}\epsilon_X^E=0.$$

Observing that $y^*(0) = y^*(\infty)$ ensures that this equation has a solution and that y^* is a hump-shaped function of τ.

7.5 Proof of Proposition 4

Taking logs of (24) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \frac{1}{\epsilon - 1} \log N(t).$$

Using the definition $n \equiv Ne^{-\lambda t}$, the expression for g^* in (25) and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$, I obtain

$$\log T(t) = \theta \log Z_0 + g^* t + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^*\right] ds + \frac{1}{\epsilon - 1} \log n(t).$$

Using (18), (23) and the definition of Δ I rewrite the third term as

$$\begin{aligned}
\theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \int_0^t \left(\frac{y^*}{n(s)} - \frac{y^*}{n^*}\right) ds \\
&= \gamma \int_0^t \left(\frac{n^*}{n(s)} - 1\right) ds \\
&= \gamma\Delta \int_0^t e^{-\nu s} ds \\
&= \frac{\gamma\Delta}{\nu}\left(1 - e^{-\nu t}\right),
\end{aligned}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\theta(\epsilon - 1)}{\epsilon} \frac{\phi\alpha - \rho}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - \rho\beta}.$$

Using (23) and the definition of Δ I rewrite the last term as

$$\begin{aligned}
\frac{1}{\epsilon - 1} \log n(t) &= \frac{1}{\epsilon - 1} \log \frac{n^*}{1 + \Delta e^{-\nu t}} \\
&= \frac{1}{\epsilon - 1} \log n_0 + \frac{1}{\epsilon - 1} \log \frac{\frac{n^*}{n_0}}{1 + \Delta e^{-\nu t}} \\
&= \frac{1}{\epsilon - 1} \log n_0 + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta}{1 + \Delta e^{-\nu t}}.
\end{aligned}$$

These results yield (26). Taking derivatives with respect to τ and observing that

$$\frac{d\Delta}{d\tau} = \frac{dy^*}{d\tau} \frac{1}{y_0} > 0$$

yields $\frac{d \log T(t)}{d\tau} > 0$.

7.6 Proof of Proposition 5

Observe that

$$\frac{d}{d\tau}\left(\log\frac{y^*}{c^*}\right) < 0 \Leftrightarrow \frac{dy^*}{d\tau} < y^*\frac{S_X^E}{P_E+\tau}$$

since

$$\frac{d\log c^*}{d\tau} = \frac{dC_X}{d\left(P_E+\tau\right)}\frac{P_E+\tau}{C_X}\frac{d\left(P_E+\tau\right)}{d\tau}\frac{1}{P_E+\tau} = \frac{S_X^E}{P_E+\tau}.$$

Now use (33), (11) and $Y = Ly^*$ to rewrite the second inequality as

$$\begin{aligned}
\frac{E}{L}\left[1-\frac{\tau}{P_E+\tau}\epsilon_X^E\right] &< \frac{E}{L}\frac{1-\beta\left(\rho-\lambda\right)}{\frac{\epsilon-1}{\epsilon}} \\
1-\frac{1}{\epsilon}-\frac{\epsilon-1}{\epsilon}\frac{\tau}{P_E+\tau}\epsilon_X^E &< 1-\beta\left(\rho-\lambda\right) \\
-\frac{\epsilon-1}{\epsilon}\frac{\tau}{P_E+\tau}\epsilon_X^E &< \frac{1}{\epsilon}-\beta\left(\rho-\lambda\right).
\end{aligned}$$

The right-hand side of this expression is positive because $\frac{1}{\epsilon}-\beta\left(\rho-\lambda\right) > 0$ since the feasibility constraint $\frac{1}{\epsilon}-\beta\rho > 0$ holds. The left-hand side is negative. Therefore, y^*/c^* is decreasing in τ for all $\tau \geq 0$.

7.7 Proof of Proposition 6

I first use (26) and the definition of Δ to write (3) as

$$\begin{aligned}
\log u^*\left(t\right) &= \log\frac{y^*}{P_Y\left(t\right)} \\
&= \log\frac{\epsilon-1}{\epsilon}+\log\frac{y^*}{c^*}+\log T\left(t\right) \\
&= \log\left(\frac{\epsilon-1}{\epsilon}Z_0^\theta n_0^{\frac{1}{\epsilon-1}}\right)+\log\frac{y^*}{c^*}+g^*t \\
&\quad +\frac{1}{\epsilon-1}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}+\frac{\gamma\Delta}{\nu}\left(1-e^{-\nu t}\right).
\end{aligned}$$

Without loss of generality, I set

$$\frac{\epsilon-1}{\epsilon}Z_0^\theta n_0^{\frac{1}{\epsilon-1}} = 1$$

and obtain (27). I then substitute this expression into (1) and write

$$\begin{aligned} U^* &= \int_0^\infty e^{-(\rho-\lambda)t}\left[\log\frac{y^*}{c^*}+g^*t\right]dt \\ &\quad+\frac{\gamma\Delta}{\nu}\int_0^\infty e^{-(\rho-\lambda)t}\left(1-e^{-\nu t}\right)dt \\ &\quad+\frac{1}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}dt. \end{aligned}$$

The first and second integrals have straightforward closed form solutions. (The third is solvable as well, but it entails a very complicated expression containing the hypergeometric function and is not worth using since it adds no insight and does not simplify the algebra in the analysis below.) Hence, I obtain (28).

7.8 Proof of Proposition 7

Let $y_0 = y^*(0)$ denote y^* evaluated at $\tau = 0$. Observe then that:

$$\Delta(0) = \frac{y^*(0)}{y_0} - 1 = 0;$$

$$\Delta(\infty) = \frac{y^*(\infty)}{y_0} - 1 = \frac{\frac{\epsilon-1}{\epsilon}}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon}} > 0;$$

$$U^*(0) = \frac{1}{\rho-\lambda}\left[\log\frac{y^*(0)}{c^*(0)}+\frac{g^*}{\rho-\lambda}\right] > 0;$$

$$\begin{aligned} U^*(\infty) &= \frac{1}{\rho-\lambda}\left[\log\frac{y^*(\infty)}{c^*(\infty)}+\frac{g^*}{\rho-\lambda}+\frac{\gamma\Delta(\infty)}{\rho-\lambda+\nu}\right] \\ &\quad+\frac{1}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\log\frac{1+\Delta(\infty)}{1+\Delta(\infty)e^{-\nu t}}dt \\ &= -\infty. \end{aligned}$$

Note that the restriction $U^*(0) > 0$ makes sense (and does not require special assumptions beyond $\log\frac{y^*(0)}{c^*(0)} > 0$), while $U^*(\infty) = -\infty$ follows from that fact that $y^*(\infty)$ is finite and $c^*(\infty) = \infty$. Next, use the properties derived in Proposition 6 and note that $\frac{dU^*}{d\tau} > 0$ iff

$$\frac{d}{d\tau}\left(\log\frac{y^*}{c^*}\right)+\frac{\gamma}{\rho-\lambda+\nu}\frac{d\Delta}{d\tau}+\frac{\rho-\lambda}{\epsilon-1}\int_0^\infty\frac{e^{-(\rho-\lambda)t}}{1+\Delta}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}\frac{d\Delta}{d\tau}dt > 0.$$

Now observe that $U^*(\infty) = -\infty$ means that the function U^* eventually must be decreasing. Moreover, the second and third terms in the inequality above are always positive. The first term is negative. It follows that $\frac{dU^*}{d\tau}$ changes sign exactly once. Therefore, U^* is hump-shaped in τ iff the inequality holds in a neighborhood of $\tau = 0$, otherwise it is always decreasing in τ.

Now observe that $\frac{dU^*}{d\tau}\mid_{\tau=0} > 0$ requires

$$\begin{aligned} \frac{d\log c^*}{d\tau} &< \left(\frac{1}{1+\Delta} + \frac{\gamma}{\rho-\lambda+\nu}\right)\frac{d\Delta}{d\tau} \\ &\quad + \frac{\rho-\lambda}{\epsilon-1}\frac{1}{1+\Delta}\frac{d\Delta}{d\tau}\int_0^\infty e^{-(\rho-\lambda)t}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}dt, \end{aligned}$$

which one can rewrite as

$$\frac{\frac{d\log c^*}{d\tau}}{\frac{d\log y^*}{d\tau}} < 1 + \frac{\gamma}{\rho-\lambda+\nu} + \frac{\rho-\lambda}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}dt$$

since

$$\frac{d\Delta}{d\tau}\frac{1}{1+\Delta} = \frac{d\log y^*}{d\tau}.$$

Letting $\tau \to 0$,

$$\frac{\frac{\epsilon}{\epsilon-1}}{y^*(0)} < 1 + \frac{\gamma}{\rho-\lambda+\nu} + \frac{\rho-\lambda}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\left(1-e^{-\nu t}\right)dt.$$

This yields

$$\frac{\epsilon}{\epsilon-1}\left[\frac{1}{\epsilon} - \beta(\rho-\lambda)\right] < \frac{\gamma + \frac{\nu}{\epsilon-1}}{\rho-\lambda+\nu},$$

which I can rewrite

$$-\frac{\epsilon}{\epsilon-1}\beta(\rho-\lambda) < \frac{\gamma(\epsilon-1)-\rho+\lambda}{\rho-\lambda+\nu}.$$

This inequality holds if

$$\gamma(\epsilon-1) > \rho-\lambda.$$

Using the definitions of γ and g^*, and a little bit of algebra, this inequality reduces to

$$g^* > \rho\frac{1-\theta(\epsilon-1)}{\epsilon-1},$$

which I can rewrite

$$\frac{g^*}{\rho} + \theta > \frac{1}{\epsilon-1}.$$

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